Final Exam. (Int. to Nuc. Eng.)
June 10(Mon), 2024. 09:00-10:00

*** Describe briefly the followings: (3 each)**

1. 우라늄 농축방법
2. 핵잠수함이 Intermediate reactor를 쓰는 이유
3. Transcendental equation
4. non-1/v factor
5. Reflector saving
6. Back end fuel cycle
7. Thermal disadvantage factor
8. Self shielding effect

*** Discuss the physical meanings: (5 each)**

9 대형원자로가 2차측을 Multi-loop로 구성하는 이유

10. Prompt jump approximation
11. Explain the influence of reflector on thermal and fast fluxes.
12. 6 factor formula
13. Quasi-homogeneous reactor vs. Heterogeneous reactor
14. Bucking B_n 의 의미
15. Draw the schematic of PWR nuclear power plant and explain
16. 3 major types of gamma ray interactions with matters.

*** Do as directed;**

17. Derive neutron flux for a finite cylindrical reactor of radius R and height H with power P. Neglect the extrapolated distance. (15)

18. Derive for a spherical reactor of radius R reflected by an infinite medium (15)

$$BR\cot BR - 1 = -\frac{D_r}{D_c}\left(\frac{R}{L_r} + 1\right)$$

19. Maximum to Average Flux and Power, Ω (10)

핵공학개론1 - 2024 final solution

1. 우라늄 농축방법
1. Gaseous Diffusion (기체확산법): 육불화우라늄(UF_6) 기체상태에서 U-235와 U-238의 원자량이 다르므로, 확산속도도 다른 것을 이용하여 농축.
2. Gas Centrifuge (원심분리법): 원자량이 다르므로 무거운 원소가 원심분리시 바깥쪽으로 더 쏠리게 됨. 이것을 이용하여 분리.

2. 핵잠수함이 Intermediate reactor를 쓰는 이유
Double Humped Curve에서 중성자의 에너지가 더 클수록 중간의 valley가 올라오는 경향을 보인다. 이렇게 올라옴으로 인해서 중성자 독물질 (Xe)의 생성량이 줄어들고, 이것은 원자로의 출력변화시 안정성에 영향을 준다. 또한 핵잠수함의 경우 노심이 작으므로, 열중성자 에너지 준위까지 감속시킬 Geometry 가 나오지 않으므로, 어쩔 수 없이 Intermediate range 를 씀.

3. Transecental Equation
L22. 18번 중성자 분포식 - Relation of Reactor core's radius and Reflector's radius의 상관관계식 처럼 삼각함수 (Trigonometric Function) 와 다항함수 (Polynomial Function) 이 같이 섞여 있는 식을 의미한다. 이때 분석적 해 (Analytical Solution)을 구하기 힘드므로, 그래프를 이용해서 해를 구함. (Graphical Solution)

4. Non 1/v factor
중성자의 흡수단면적은 중성자 속도에 반비례하는 영향을 보인다. 하지만 노심에서 중요하게 쓰이는 물질 (U-238, Zr) 들이 정확하게 반비례하는 경향을 보이지 않으므로, 이러한 물질들에 대해서 보정해주는 인자를 Non 1/v factor (g) 라고 한다.

5. Reflector Saving
L23. Decrease in the critical dimensions of a reactor core as a result of the use of reflector. The equation:

$$\delta = R_0 - R$$

Where δ is the Reflector Saving, R_0 is the reactor size when not using the reflector, and R is the size when using reflector.
반사체를 사용함으로 얻는 Reactor 크기의 차이.

6. Back end fuel cycle L11. From withdrawl of fuel to final disposition
핵연료를 원자로에서 제거한 시점부터 폐기처리 까지의 과정. 핵연료 재처리, 방폐장에 저장 등을 포함한다.

7. Thermal disadvantage Factor
L25, page 5: The Thermal Disadvantage factor 'zeta' is

$$\zeta = \frac{\overline{\phi}_{TM}}{\overline{\phi}_{TF}}$$

Where the $\overline{\phi}_{TM}$ is the thermal flux at the moderator, and $\overline{\phi}_{TF}$ is the thermal flux at the fuel. Why is this a disadvantage? Because the more thermal flux in the moderator, the less thermal neutron will be effectively used for the fission.

8. Self shielding effect
L25. The flux is lower in the fuel than it is in the moderator. This depression in the flux is caused by the fact that some of the enutrons entering the fuel from the moderatior are absorbed near the surface of the fuel. This is called "Self Shielding".
In the 4 factor formula, the Thermal Utilization factor (Fuel utilization factor, f) is larger in case of Quasi-homogeneous reactor compared to heterogeneous reactor (Q13). This is caused because the flux distribution in the moderator is higher than that of the fuel. The flux entering the fuel is absorbed near the surface of the fuel, resulting even less flux at the center of the fuel.
+ 자기차폐. Energy Self Shielding 과 Spatial Self Shielding이 있다.

9. 대형원자로가 2차측을 Multi-loop로 구성하는 이유
Redundancy: APR1400 / OPR1000 같은 경우 SG가 2개 / RCP 4개. Westinghouse 노형중에는 SG가 3, 4개 인것도 있다. 이 이유는 하나의 부품(SG든 RCP든 FWP든) 에 문제가 생기더라도, 나머지 하나로 어느정도의 노심 냉각을 달성하려 하기 위함이다.

10. Prompt Jump Approximation
몰?루? 자료에 없던거 같은데
Prompt Jump Approximation -This approximation assumes:
1. The change in neutron population due to prompt neutrons is instantaneous.
2. Delayed neutron precursors don't change appreciably during the jump.

3. After the jump, the system resumes normal kinetics dominated by both prompt and delayed neutrons.

11. Explain the influence of reflector and fast fluxes.

Reflector라고 해서 중성자를 반사시키는게 아니라, Fast neutron을 감속시켜 열중성자로 만드는 감속재를 Reactor Vessel의 periphery에 두는게 Reflector이다. 가장자리 부분의 thermal flux를 높임으로서 코어 내부의 flux distribution을 조금 더 평탄하게 만들어준다. 이렇게 되면 thermal flux의 leak (which is dependent on $D\nabla\phi$) 가 줄어들게 된다.

12. 6 factor formula 핵공1에서는 이렇게 배우는데

$$\frac{k_\infty}{(1+B^2{L_T}^2)(1+B^2\tau_T)} = 1 \quad \text{where} \quad {L_T}^2 = \frac{\overline{D}}{\overline{\Sigma}_a}, \quad \tau_T = \frac{D_1}{\Sigma_1}$$

근데 이걸 결국에 전개하면 노이론1에서 배우는 식이 나옴.

$$k = pf\eta\varepsilon L_f L_t$$

- p: Resonance escape probability: 중성자가 높은 흡수단면적을 가지고 있는 공명영역을 뚫고 넘어올 확률
- f: Thermal Utilization Factor: 핵연료에서 중성자가 흡수될 확률을 원자로 전체에서 중성자가 흡수될 확률로 나눈 값. $f = \frac{\Sigma_a^F}{\Sigma_a}$
- η: Thermal Reproduction: 열중성자가 핵연료에 흡수되었을때 핵분열로 생성되는 중성자 수의 비. $\eta = \frac{\nu\Sigma_f^F}{\Sigma_a^F}$
- ε: Fast Fission Factor, 전 에너지 영역 중성자가 일으키는 핵분열중성자의 수와 열중성자가 일으키는 핵분열중성자 수의 비. $\frac{fast+thermal}{thermal}$
- L_f or P_F: Fast Leakage: 속중성자 영역의 중성자가 **원자로 외부로 유출되지 않을 확률**.
- L_t or P_T: Thermal Leakage: 열중성자 영역의 중성자가 원자로 외부로 유출되지 않을 확률.

13. Quasi-homogeneous reactor vs. Heterogeneous reactor

만약 중성자의 모든 에너지 레벨에 대해 중성자의 평균비정이 핵연료봉의 두께보다 두꺼울 때, 중성자가 핵연료봉에서 한번 이상 반응할 확률이 매우 낮다. 이때 Quasi-homogeneous reactor로 계산한다. 반면 평균비정이 핵연료봉의 크기보다 작을 경우, 중성자가 연료봉 내에서 여러번 충돌(반응)할 것이다. 이때는 Heterogeneous reactor로 계산하여야 한다.

14. Buckling B_n 의 의미

n-th eigenvalue 에 대한 buckling.
예시를 들어보자. In a bare slab reactor:, the flux will be:

$$\phi(x) = A\cos Bx \quad \xrightarrow{BC:\ \phi(\tilde{a}/2)=0} \quad \phi\left(\frac{\tilde{a}}{2}\right) = A\cos B\frac{\tilde{a}}{2} = 0$$

$$\therefore B_n = \frac{n\pi}{\tilde{a}}$$

This is an eigenvalue problem, where B_n is eigenvale and $\cos B_n x$ is called as eigenfunction.

15. Draw the schematic of PWR nuclear power plant and explain

You know the Drill, just do it

16. 3 major types of gamma ray interaction with matters

Photoelectric effect, Compton scattering, Pair production

17. Derive neutron flux for a finite cylindrical reactor of radius R and height H with power P. Neglect the extrapolated distance.

Start with the reactor equation:

$$\nabla^2\phi + B^2\phi = 0$$

We need to consider the radial direction (r) and axial direction (z). Thus the Laplacian(∇^2) becomes:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) + \frac{\partial^2\phi}{\partial z^2} + B^2\phi = 0$$

$$\text{BCs:} \quad \phi(R,z) = 0, \quad \phi(r,H/2) = 0 \quad \leftarrow \quad \text{we neglect extrapolated length}$$

We can assume that the radial and axial directions are independent from each other, i.e.:

$$\phi(r,z) = R(r)\cdot Z(z)$$
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial R}{\partial r}\right) + {B_r}^2 R = \frac{\partial^2 Z}{\partial z^2} + {B_z}^2 Z = 0$$
$$\therefore \phi(r,z) = AJ_0\left(\frac{2.405r}{R}\right)\cos\left(\frac{\pi z}{H}\right)$$

18. spherical reactor에서 다음 식을 유도하여라:

$$BR\cot BR - 1 = -\frac{D_r}{D_c}\left(\frac{R}{L_r}+1\right)$$

L22 에서 확인
For the core, the reactor equation is:

$$\nabla^2\phi_c + B^2\phi_c = 0 \quad \rightarrow \quad \phi_c = A\frac{\sin Br}{r}$$

For the reflector, where there is no neutron generation, relies only on diffusion mode:

$$\nabla^2\phi_r - \frac{1}{{L_r}^2}\phi_r = 0 \quad \rightarrow \quad \phi_r = A'\frac{e^{-r/L_r}}{r}$$

At the boundary (edge of the core = starting point of the core shroud) r = R:

$$\phi_c(R) = \phi_r(R) \quad \rightarrow \quad A\frac{\sin BR}{R} = A'\frac{e^{-R/L_r}}{R} \tag{1}$$

Also, the current (J) should be same at the boundary:

$$J_c(R) = J_r(R) \quad \rightarrow \quad -D_c\nabla\phi_c(R) = -D_r\nabla\phi_r(R)$$
$$AD_c\left(\frac{B\cos BR}{R} - \frac{\sin BR}{R^2}\right) = A'D_r\left(-\frac{e^{-R/L_c}}{RL_r} + \frac{e^{-R/L_c}}{R^2}\right) \tag{2}$$

In order to get a nontrivial solution, the determinants of the coefficient should be zero. This means Dividing Eq.2 with Eq.1:

$$D_c\left(B\cot BR - \frac{1}{R}\right) = D_r\left(\frac{1}{L_r}+\frac{1}{R}\right)$$
$$\rightarrow \quad \underline{\boldsymbol{BR\cot BR - 1 = -\frac{D_r}{D_c}\left(\frac{R}{L_r}+1\right)}}$$

19. Maximum to Average Flux and Power, Ω L23, pg 7
Reflector's work is to flatten out the flux distribution. This also means that not only the reflector reduces the critical size and mass of the reactor (Q5: Reflector Saving), but also <u>reduces the maximum to average flux ratio</u>. This can be represented as:

$$\Omega = \frac{\phi_{max}}{\overline{\phi}}$$

The $\overline{\phi}$ can be acquired by integrating the flux over the reactor domain.
나중에 원자로이론1 에서 Flux Peaking Factor, f 로 배움.

Final Exam. (Int. to Nuc. Eng.)
June 14(Thu), 2022. 12:00-13:15

*** Describe briefly the followings: (3 each)**

1. Wet steam 액체를 포함한 습증기
2. Thermal neutron 0meV~0.5eV 사이의 에너지를 갖는 neutron
3. Transcendental equation 초월 방정식. 미지수의 대수식만으로는 나타낼 수 없는 방정식.
4. HTGR High Temperature Gas-Cooled Reactor.
5. Laplacian ∇^2, The divergence of the gradient of a scalar function
6. Back end fuel cycle 후행 핵주기. 원자로에서 꺼내진 이후 과정.

*** Discuss the physical meanings: (5 each)**

7. Double humped curve fission product가 균등한 질량수로 쪼개지지 않음.
8. Meaning of Buckling 원자로에서 누설되는 중성자 속의 곡률을 나타내는 값.
9. Explain the influence of reflector on thermal and fast fluxes.
   Thermal flux는 core와 reflector 경계 가까이에서 상승하고 reflector 내부에서 peak 값을 가진다.
   ↳ reflector는 원자로의 임계 크기 및 질량 감소.
10. 6 factor formula $\frac{k_\infty}{(1+B^2L_T^2)(1+B^2\tau_T)}$

*** Do as directed;**

11. Calculate neutron flux in an infinite slab of thickness *2a* with an infinite planar source at its center emitting S neutrons per cm²/sec. The neutron flux banishes at the extrapolated surfaces of the slab. (15)
$\frac{SL}{2D}\frac{\sinh[(a+d-|x|)/L]}{\cosh[(a+d)/L]}$ $)=0$ $A=\frac{S_0}{2D}$

12. Derive neutron flux for a finite cylindrical reactor of radius R and height H with power P. Neglect the extrapolated distance. (15) $\to R(r)=AJ_0(\frac{2.405r}{R})$ $\to Z(z)=A'\cos B_z z$
$\phi(r,z)=R(r)Z(z)$, $\frac{1}{r}\frac{d}{dr}r\frac{d(R(r)Z(z))}{dr}+\frac{d^2(R(r)Z(z))}{dz^2}+B^2(R(r)Z(z))=0$ $=\frac{1}{R(r)}\frac{1}{r}\frac{d}{dr}r\frac{dR(r)}{dr}+\frac{1}{Z(z)}\frac{d^2Z(z)}{dz^2}+B^2=0$ $\therefore \phi=CJ_0(\frac{2.405r}{R})\cos(\frac{\pi z}{H})$

13. A bare spherical thermal reactor, 100 cm in radius, consists of a homogeneous mixture of ^{235}U and graphite. The reactor is critical and operates at a power level of 100 kWth. Calculate followings;
   (a) buckling (5) 3.14×10^{-2} (b) k_∞ (10) 1.8 (c) critical mass (5)
   η_T = 2.065, σ_{aM} = 0.0034 *b*, σ_{aF} = 681 *b*
   ρ = 1.60 g/cm³, $\overline{D}$ = 0.84 cm, $\overline{\Sigma}_a$ = 2.4× 10^{-4} cm^{-1}, L_T^2 = 3500 cm²
   D_1 = 1.016 cm, Σ_1 = 0.00276 cm^{-1} , τ_T = 368 cm²

bare spherical $B^2=(\frac{\pi}{R})^2$

$k_\infty=\eta_T f=1.8$

$f=\frac{Z}{Z+1}=0.87$

$Z=\frac{1+B^2(L_T^2+\tau_T)}{\eta_T-1-B^2\tau_T}=6.87$

$M_F=\frac{N_F V}{N_A}\times M_F$

$=N_F\times\frac{VM_F}{N_A}$

$=Z\frac{\Sigma_{aM}}{\sigma_{aF}}\frac{N_A}{\rho V M_m}\times\frac{VM_F}{N_A}$

$\frac{3\tau Z}{1+Z+b^2(L_T^2+Z\tau_T)}$

$M_M=\rho V=1.6\times\frac{4}{3}\pi R^3$

$M_F=Z\frac{\sigma_{aM}M_m}{\sigma_{aF}M_m}M_F=4.5$

$N_F=Z\frac{\sigma_{aM}}{\sigma_{aF}}\frac{M_n}{M_n}$

14. Maximum to Average Flux and Power, Ω (10)
$\Omega=\frac{\phi_{max}}{\phi_{av}}$ $\phi_{av}=\frac{P}{E_R\Sigma_f V}$

15. Why $f_{hetero} < f_{homo}$? (10)
$f=\frac{\Sigma_{aF}V_F\phi_{TF}}{\Sigma_{aM}V_M\phi_{TM}+\Sigma_{aF}V_F\phi_{TF}}$ $\frac{\Sigma_{aF}V_F}{\Sigma_{aM}V_M\zeta+\Sigma_{aF}V_F}$
ζ_{Tn} $\therefore f_{homo}>f_{hetero}$
homogeneous ⇒ $\zeta=1$
heterogeneous ⇒ self shielding 때문에 $\phi_{TF}<\phi_{TM}$, $\zeta>1$.

2022 기말 solution

1. Wet steam 습증기. 물 (droplet)을 포함한 증기

2. Thermal Neutron 열중성자. 0.0253 eV, $V = 2200\ m/s$

3. Transcendental equation 초월방정식. 삼각함수와 다항함수가 섞여있는 함수로서 분석적 해 (analytical solution)을 구하기 힘들고, 그래프 혹은 수치적으로 풀이 (graphical solution)

4. HTGR High Temperature Gas-cooled Reactor

5. Lapacian

$$\text{div}(grad\ f) = \nabla \cdot (\nabla f) = \nabla^2 f$$

6. Back end fuel cycle. 후행핵주기. 원자로에서 withdrawl 후 처분까지의 과정

7. Double humped curve 이건 알잖아

8. Meaning of Buckling
In a Reactor equation:

$$\nabla^2 \phi + B^2 \phi = 0$$

The Buckling, B^2 means the convexity of the flux distribution. The greater the value of B^2,the larger the convexivity and leakage on the bounary.

9. Explain the influence of reflector on thermal and fast fluxes.
Reflector moderates the fast flux into thermal. This makes the thermal flux distribution on the edge to go up, making the curve flat. This decreases the neutron leak.

10. 6 factor formula

$$\frac{k_\infty}{(1 + B^2 {L_T}^2)(1 + B^2 \tau_T)}$$

Its physical meaning? k_∞ is a criticality of infinite reactor. In there, we multiply the Fast non-leakage probability(L_f or P_F) and thermal non-leak probability(L_t or P_T). This way, we can calculate the criticality of finite reactor. (2 group, obviously)

11. Calculate the neutron flux in a finite slab of thickness 2a with an infinite planar source at its center emitting S neutrons per cm^2/s. The neutron flux banishes at the extrapolted surface of the slab.
The diffusion equation:

$$\nabla^2 \phi - \frac{1}{L^2}\phi + \frac{s}{D} = 0$$

s denotes the volumetric neutron generation rate. In this case, there is only a planar source, thus $s = 0$. Solving the above equation in cartesian coordinate, assuming the thickness direction is a:

$$\nabla^2 \phi - \frac{1}{L^2}\phi = 0 \quad \rightarrow \quad \frac{d}{dx}\frac{d\phi}{dx} - \frac{1}{L^2}\phi = 0$$

General solution for this 2nd order ODE is

$$\phi = Ae^{-x/L} + Ce^{x/L}$$

Let d the extrapolated length from the surface. Then the boundary condition is

$$\phi(a + d) = \phi(-a - d) = 0$$
$$\phi(a + d) = Ae^{-(a+d)/L} + Ce^{(a+d)/L} = 0 \quad \rightarrow \quad C = -Ae^{-2(a+d)/L}$$

Also, the single-surface current at $x=0$ should be $S/2$. This gives

$$\lim_{x\to 0+} J(x) = S/2, \quad J(x) = -D\nabla\phi(x) = \frac{S}{2}.$$
$$-D\frac{d\phi(x)}{dx}\bigg|_{x=0} = S/2 \quad \to \quad -D\left(-\frac{A}{L}e^{-x/L} + \frac{C}{L}e^{x/L}\right)\bigg|_{x=0} = S/2$$
$$\therefore \quad A = \frac{SL}{2D}\frac{1}{1+e^{-2(a+d)/L}}$$
$$\phi = \frac{SL}{2D}\left(\frac{1-e^{-2(a+b)/L}}{1+e^{-2(a+b)/L}}\right)e^{-x/L}$$

This should be symmetrical to $x=0$. Giving absolute value to the x:

$$\phi = \frac{SL}{2D}\left(\frac{1-e^{-2(a+b)/L}}{1+e^{-2(a+b)/L}}\right)e^{-|x|/L}$$

12. Derive neutron flux for a finite cylindrical reactor of radius R and H with power P. Neglect the extrapolated distance.
Starting from reactor equation:

$$\nabla^2\phi + B^2\phi = 0$$

We are given finite cylinder. We should assume that the flux is independent from each r and z component:

$$\phi(r,z) \equiv R(r)\cdot Z(z), \quad B^2 = {B_r}^2 + {B_z}^2$$
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial R}{\partial r}\right) + {B_r}^2 R = \frac{\partial^2 Z}{\partial z^2} + {B_z}^2 Z = 0$$

The general solution for each cases are:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dR}{dr}\right) + {B_r}^2\phi = \frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr} + {B_r}^2 = 0$$
$$R = AJ_0\left(\frac{2.405r}{R}\right)$$
$$Z = C\cos\left(\frac{\pi z}{H}\right)$$
$$\therefore \quad \phi = AJ_0\left(\frac{2.405r}{R}\right)\cos\left(\frac{\pi z}{H}\right)$$

With the Power of P, We can get the coefficient A:

$$P = E_R\Sigma_f\int_{V}\phi\, dV = E_R\Sigma_F A\int_{r=0}^{R} J_0\frac{2.405r}{R}\, rdr \times \int_{z=-H/2}^{H/2}\cos\left(\frac{\pi z}{H}\right)\, dz \times \int_{\theta=0}^{2\pi} d\theta$$
$$\cdots \quad A = \frac{3.63P}{VE_R\Sigma_f}, \qquad \underline{\boldsymbol{\phi(r,z) = \frac{3.63P}{VE_R\Sigma_f}J_0\left(\frac{2.405r}{R}\right)\cos\left(\frac{\pi z}{H}\right)}}$$

More derivation:
We have to find the derivative of the Bessel function. We'll use the following formula for the derivative of $J_\nu(x)$:

$$\frac{d}{dx}J_n(x) = J_{n-1}(x) - \frac{n}{x}J_n(x)$$
$$\frac{d}{dx}x^nJ_n(x) = nx^{n-1}J_n(x) + x^n\frac{d}{dx}J_n(x) \quad \text{(product rule)}$$
$$= nx^{n-1}J_n(x) + x^n\left(J_{n-1}(x) - \frac{n}{x}J_n(x)\right) = x^nJ_{n-1}(x)$$

Thus

$$\frac{d}{dx}x^\nu J_\nu(x) = x^\nu J_{\nu-1}(x)$$
$$\text{or} \quad \int x^\nu J_{\nu-1}(x)\, dx = x^\nu J_\nu(x)$$

In this case, $\nu = 1$. thus

$$\int xJ_0(x)dx = xJ_1(x) + C \quad \rightarrow \quad \int xJ_0(f(x))dx = \frac{x}{f'(x)}J_1(f(x)) + C$$

Then the integration becomes

$$2\pi\left[\frac{R}{2.405}rJ_1(\frac{2.405r}{R})\right]_0^R \times \frac{2H}{\pi} = 2\pi\frac{R^2}{2.405}J_1(2.405)\times\frac{2H}{\pi}$$

The value for Bessel equation of first kind:

$$J_1(2.405) = 0.5191 \quad \text{matlab: besselj(1,2.405)}$$

Thus plugging all these shits in we get

$$P = AE_r\Sigma_f 4\pi R^2 H\frac{0.5191}{2.405\pi}$$

$$A = \frac{P}{E_r\Sigma_f \overset{V}{\cancel{\pi R^2 H}} \times 0.27482} = \frac{3.638P}{E_r\Sigma_f V}$$

Finally

$$\phi(r,z) = \frac{3.638P}{E_r\Sigma_f V}J_0\left(\frac{2.405rr}{R}\right)cos\left(\frac{\pi z}{H}\right)$$

■

---

13. A bare spherical thermal reactor, 100 cm in radius, consists of a homogeneous mixture of ^{235}U and graphite. The reactor is critical and operates at a power level of 100 kWth. Calculate the following: 1. Buckling, 2. k_∞, 3 critical mass

14. Maximum to Average Flux and Power, Ω

$$\Omega = \frac{\phi_{max}}{\phi_{avg}}$$

The average flux is obtained by integrating the flux distribution in whole domain and dividing with the volume.

15. Why $f_{hetero} < f_{homo}$?
f here is fuel utilization factor, where

$$f = \frac{\Sigma_a^F}{\Sigma_a}$$

In case of homogeneous reactor, the moderator and fuel are mixed alltogether - meaning it will have a even distribution. However, in case of heterogeneous reactor, the flux tends to be dip at the center of the fuel, because some of the neutrons that are entering the fuel are absorbed at the surface, since the fuel's absorption XS is much higher than that of moderator. This is called "Self Shielding" - more specifically, Spatial Self Shielding.

Plus. Infinite medium 에서 중성자의 diffusion length는 중성자의 평균 흡수 거리의 $1/\sqrt{6}$ 임을 보여라.
We design a equation to see the change in number of neutrons as they move through a medium, at a distance between r and $r + dr$:

$$dn = \Sigma_a\phi(r)\ d\mathcal{V}$$

For ϕ, we will use point source case, and $d\mathcal{V}$ will be $4\pi r^2\ dr$:

$$\phi(r) = \frac{Se^{-r/L}}{4\pi Dr}$$

$$dn = \frac{Se^{-r/L}}{4\pi Dr}\ 4\pi r^2 dr = \frac{S\Sigma_a}{D}re^{-r/L} = \frac{S}{L^2}re^{-r/L}\ dr$$

This means that the number of neutrons that will survive through dr is dn. If we divide this with S, we will get

the survival probability:

$$p(r)dr = \frac{1}{L^2} r e^{-r/L}\ dr$$

for somewhat obscure reason (textbook literally says this), it is more usual in nuclear engineering to compute the average of the square of the distance:

$$\overline{r^2} = \int_0^\infty r^2 p(r)\ dr = \int_{r=0}^\infty \frac{1}{L^2} r^3 e^{-r/L}\ dr$$
$$= \frac{1}{L^2}\left[-r^3 L e^{-r/L} - 3r^2 L^2 e^{-r/L} - 6rL^3 e^{-r/L} - 6L^4 e^{-r/L}\right]_{r=0}^\infty = 6L^2$$
$$\underline{\boldsymbol{L = \frac{1}{\sqrt{6}}\overline{r}}}$$

Group diffusion equation

$$-D_g \nabla^2 \phi_g + \Sigma_{a,g}\phi_g + \sum_{h=g+1}^{N} \Sigma_{s,g\to h}\phi_g = \sum_{h=1}^{g-1} \Sigma_{s,h\to g}\phi_h + s_g$$

Each term means:
$-D_g \nabla^2 \phi_g$: Amount of leak happening in group g
$\Sigma_{a,g}\phi_g$: Amount of absorbed neutron in group g
$\sum_{h=g+1}^{N}$: Amount of neutron that is scattered to lower energy level (lowest energy level: N)
$\sum_{h=1}^{g-1} \Sigma_{s,h\to g}\phi_h$: Amount of neutron scattered into current energy group from higher energy group
s_g: Amount of neutron (fission, delayed, etc) created at energy group g. This is also annotated as $\chi_g \sum_{h=1}^{N} \nu\Sigma_{f,h}\phi_h$

Final Exam. (Int. to Nuc. Eng.)
June 8(Tue), 2021. 15:00-16:00

∗ Write answer in the same color of the problem (3k, 3k+1, 3k+2)

∗ Describe briefly the followings: (3 each)

1. non-1/v factor 1/v 흡수거동하지 않는 흡수체에
2. Ω $\frac{\phi_{max}}{\phi_{av}}$
3. Godiva Bare Reactor 대해 보정하는 보상인자.
4. Double humped curve 핵분열생성물이 균등하지 않게 갈라지는 것을 보여주는 곡선.
5. Reflector saving
   Reflector를 사용함으로써 원자로심의 임계크기를 줄이는 것.
6. Thermal disadvantage factor
   $$\zeta = \frac{\bar{\phi}_{TM}}{\bar{\phi}_{TF}}$$

∗ Discuss the physical meanings

7. Quasi-homogeneous reactor vs. Heterogeneous reactor (6)
   Quasi-homogeneous : 노심두께가 중성자의 MFP보다 작아서 2회 이상의 연속적인 충돌을 일으킬 확률 거의 X.
   Heterogeneous : 노심두께가 중성자의 MFP보다 커서 연료봉 내에서 여러 번의 충돌을 일으킬 수 있음.
8. Self shielding effect (5) 핵연료의 표면에서 중성자가 흡수되어 핵연료 내부에서의 중성자속은 증가시키지 못함.
9. Influence of reflector on thermal and fast fluxes (5) Thermal flux는 노심과 반사체 경계 근처에서 상승하고 반사체 내부에서 Peak값을 가진다.
10. Neutron economy of the heterogeneous and homogeneous reactor (5)
    heterogeneous reactor가 homogeneous reactor보다 중성자경제성↑.
    $(fp)_{hetero} > (fp)_{homo}$, $\epsilon_{hetero} > \epsilon_{homo}$,
    $\therefore k_{\infty\,hetero} > k_{\infty\,homo}$.
11. 6 Factor formula and meaning of each term (8) $\frac{k_\infty}{(1+B^2L_T^2)(1+B^2\tau_T)}$
12. Bessel function (7) $\frac{d^2\phi}{dr^2} + \frac{1}{r}\frac{d\phi}{dr} + \left(B^2 - \frac{m^2}{r^2}\right)\phi = 0$
13. Eigenvalue problem (7) 고유값 문제.
14. Critical equation (5) $\frac{k_\infty - 1}{L^2} = B_1^2$

∗ Do as directed;

15. Derive neutron flux for a finite cylindrical reactor of radius R and height H with power P. Neglect the extrapolated distance. (20)

$$\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} + B^2\phi = 0$$

Finite ϕ as r → 0

$$\phi(\tilde{R},\ z) = 0,\ \phi(r,\ \tilde{H}/2) = 0$$

$\phi = R(r)Z(z)$로 나눔

$$\underbrace{\frac{1}{R}\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial \phi}{\partial r}}_{R\text{의 함수}} + \underbrace{\frac{1}{Z}\frac{\partial^2 \phi}{\partial z^2}}_{Z\text{함수}} + B^2 = 0$$

$$B_r^2 + B_z^2 = B^2$$

R함수 해 $= AJ_0\left(\frac{2.405r}{R}\right)$

Z함수 해 $= A'\cos\left(\frac{\pi z}{H}\right)$

$$\therefore \phi = CJ_0\left(\frac{2.405r}{R}\right)\cos\left(\frac{\pi z}{H}\right)$$

16. Derive (15)

$$BR\cot BR - 1 = -\frac{D_r}{D_c}\left(\frac{R}{L_r} + 1\right)$$

$\nabla^2\phi_c + B^2\phi_c = 0$, $\nabla^2\phi_r - \frac{1}{L^2}\phi_r = 0$

$B^2 = \frac{k_\infty - 1}{L_c^2}$

$\phi_c(R) = \phi_r(R)$

$J_c(R)n = J_r(R)n$

연립하면 $BR\cot BR - 1 = -\frac{D_r}{D_c}\left(\frac{R}{L_r} + 1\right)$

$\phi_c = A\frac{\sinh Br}{r}$ $\phi_r = A\frac{e^{-r/L}}{r}$

2021 기말 solution

여기서는 위에서 안한거만 빠르게 훑고 지나갈게요

3. Godiva

Bare reactor, where there is no reflector nor blanket. Its a critical fast reactor containing a homogeneous mixture of fuel and coolant.

10. Neutron economy of the heterogeneous and homogeneous reactor

Neutron economy is about whether the neutron is fully used for the the fission instead of other factors, such as leakage, absorption, and so on. The Thermal utilization factor f:

$$f = \frac{\Sigma_a^F}{\Sigma_a}, \quad f_{hetero} < f_{homo}$$

However, the resonance escape probability is higher for heterogeneous, and its greater than the decrease of fuel utiliztion factor:

$$(fp)_{hetero} > (fp)_{homo}$$

Also, the fast fission factor is higher in heterogeneous, since the unattenuated fast flux will have more probability of hitting the uranium, when the uranium is not surrounded by the moderator. This makes:

$$\varepsilon_{hetero} > \varepsilon_{homo}$$

Using this in 4 factor formula, $k_\infty = \eta f p \varepsilon$

$$k_{\infty,hetero} > k_{\infty,homo}$$

This means that the multiplication factor is higher in heterogeneous case compared to homogeneous case, i.e. more neutron is used for fission in heterogeneous reactor. This literally means the neutron economy.

12. Bessel Function

Bessel equation is

$$\frac{d^2\phi}{dx^2} + \frac{1}{x}\frac{d\phi}{dx} + \left(B^2 - \frac{m^2}{r^2}\right)\phi = 0$$

The solution for this ODE is called Bessel function, J and Y:

$$\phi = AJ_0(Br) + CY_0(Br)$$

13. Eigenvalue problem

In reactor equation:

$$\nabla^2\phi + B^2\phi = 0$$

여기서 slab라고 가정하면, 일반해가

$$\phi(x) = A\cos Bx + C\sin Bx$$

인데, Boundary condition을 쓴다고 해도 저 계수 (A)를 구하지 못하고, 함수 내의 계수 (B)만 구하는 것이 가능함. 또한 Boundary condition을 써서 구한 B의 값의 종류가 무한대 ($B_n = \frac{n\pi}{\tilde{a}}$) 여서, 다른 고유값 (Eigenvalue, 여기서는 B_n)에 따라 계수 (A) 도 달라짐. 이러한 문제를 고유값 문제 (Eigenvalue Problem)이라고 부름. 저기서 $\cos B_n x$ 은 고유함수 (Eigenfunction)이라고 부른다.

14. Critical equation

Reactor equation:

$$\nabla^2\phi + {B_c}^2\phi = 0$$

The reactor is critical when the k_∞ is 1. When critical, the buckling is

$$B_c{}^2 = \frac{\frac{\nu\Sigma_f}{\Sigma_a} - 1}{\frac{D}{\Sigma_a}} = \frac{\nu\Sigma_f - \Sigma_a}{D}$$

$$\frac{D}{\Sigma_a} = L^2, \quad \frac{\nu\Sigma_f}{\Sigma_a} \to k_\infty$$

$$\therefore \quad \nabla^2\phi + \frac{k_\infty - 1}{L^2}\phi = 0, \quad B^2 = \frac{k_\infty - 1}{L^2}$$

Rearranging the last equation gives us the **Critical Equation**:

$$\frac{k_\infty}{1 + B^2L^2} = 1$$

추가. Fick's law 가 Valid 하지 않은 경우 3가지

Fick's law is invalid when: 1. In a medium that strongly absorbs neutron
2. Within three mean free paths of either a neutron source or the surface of a material
3. When neutron scattering is strongly anisotropic
4. In a medium of low density

한국어:
1. 중성자를 잘 흡수하는 매질일때
2. 중성자 평균비정의 3배 이내이거나, 중성자원 근처이거나 물질 표면 근처일 때.
3. 중성자 산란반응이 비등방성일때
4. 매질의 밀도가 낮을때
핵공1 기준에서는 1,2,3번 쓰셈. 4번쓰면 틀림 (이건 노이론1 내용)

추가. 중성자 에너지 분포가 Maxwellian function을 따른다고 할 때 One group thermal flux를 구하라.

$$n(E) = \frac{2\pi n}{(\pi kT)^{3/2}} E^{1/2} e^{-E/kT}, \quad v_T = \left(\frac{2kT}{m}\right)^{1/2} = \left(\frac{2E}{m}\right)^{1/2}$$

By the definition of flux:

$$\phi(E) = n(E)v(E) = \frac{2\pi n}{(\pi kT)^{3/2}} E^{1/2} e^{-E/kT} \cdot \left(\frac{2E}{m}\right)^{1/2} = \frac{2\pi n}{(\pi kT)^{3/2}} E \left(\frac{2}{m}\right)^{1/2} e^{-E/kT}$$

The total flux is

$$\begin{aligned}\phi_T = \int_E \phi(E)\ dE &= \frac{2\pi n}{(\pi kT)^{3/2}} \left(\frac{2}{m}\right)^{1/2} \int_{E=0}^{\infty} E e^{-E/kT}\ dE \\ &= \frac{2\pi n}{(\pi kT)^{3/2}} \left(\frac{2}{m}\right)^{1/2} \cdot (kT)^2 = \frac{2n}{\sqrt{\pi}} \left(\frac{2kT}{m}\right)^{1/2}\end{aligned}$$

Using v_T:

$$\phi_T = \frac{2n}{\sqrt{\pi}} \left(\frac{2kT}{m}\right)^{1/2} = \frac{2}{\sqrt{\pi}} n v_T$$

Using $\phi_0 = nv_0$:

$$\frac{\phi_0}{\phi_T} = \frac{\sqrt{\pi}}{2} \frac{v_0}{v_T}$$

From relation:

$$E = \frac{1}{2}mv^2 = kT \quad \to \quad v \propto \sqrt{T}$$

Therefore

$$\frac{\phi_0}{\phi_T} = \frac{\sqrt{\pi}}{2} \left(\frac{T_0}{T_T}\right)^{1/2}$$